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National Institute of Technology Karnataka, Surathkal
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MA939 Functional Analysis
Problem Sheet - 10

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Answer **ALL** questions.

1. For every $p \geq 1$, prove the following statements.
   (a) ℓ_p is linearly isometric to a subspace of $L_p[0, \infty)$.
   (b) ℓ_p is linearly isometric to a subspace of $L_p[0, 1]$.
2. In $C[0, 1]$, M_1 = all constant functions, M_2 = all functions f such that $f(0) = 0$, M_3 = all functions f such that $f(0) = f(1) = 0$.
   (a) Show that M_1, M_2, and M_3 are closed subspaces of $C[0, 1]$.
   (b) Is $C[0, 1]$ the direct sum of the three subspaces?
3. Let B_1 and B_2 be the closed unit balls of $(X, \|.\|_1)$ and $(X, \|.\|_2)$ respectively. Suppose two norms $\|.\|_1$ and $\|.\|_2$ are equivalent, then prove that B_1 and B_2 are homeomorphic.
4. Prove the following statements.
   (a) Any bounded linear operator $T : c_{00} \to c_{00}$ can be represented by a column-finite infinite matrix whose entries k_{ij} are scalars with $|k_{ij}| \leq \alpha, \forall i, j \geq 1$ and some $\alpha \in \mathbb{R}$.
   (b) Any bounded linear operator $T : \ell_1 \to \ell_1$ is represented by an infinite matrix (k_{ij}) in the sense that $(Tx)_i = \sum_{j=1}^{\infty} k_{ij} x_j$ with $\|T\| = \sup_j \sum_{i=1}^{\infty} |k_{ij}|$, the supremum of the column sums of the matrix $(|k_{ij}|)$.
   (c) Any bounded linear operator $T : \ell_\infty \to \ell_\infty$ is represented by an infinite matrix (k_{ij}) in the sense that $(Tx)_i = \sum_{j=1}^{\infty} k_{ij} x_j$ with $\|T\| = \sup_i \sum_{j=1}^{\infty} |k_{ij}|$, the supremum of the row sums of the matrix $(|k_{ij}|)$.
   (d) Let X be a sequence space ℓ_p $(1 \leq p < \infty)$ or c_0. Any bounded linear operator $T : X \to X$ is represented by an infinite matrix (k_{ij}) in the sense that $(Tx)_i = \sum_{j=1}^{\infty} k_{ij} x_j$.
5. Prove or disprove. Every bounded linear operator $T : c \to c$ can be represented by an infinite matrix (k_{ij}) in the sense that for each $x \in c$, $(Tx)_i = \sum_{j=1}^{\infty} k_{ij} x_j$, the series being convergent for all i, x.