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National Institute of Technology Karnataka, Surathkal
Odd Semester, 2013 - 2014
MA939 Functional Analysis
Problem Sheet - 1

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Answer **ALL** questions.

1. Let X be a normed space. Prove that the maximum of two norms on X is again a norm.
2. Using $\varepsilon - \delta$ definition, prove that $\|.\|$ is a continuous function.
3. Using the definition of continuous function between topological spaces, prove that $\|.\|$ is a continuous function.
4. Let X be a normed space. Define jointly continuous. In terms of open sets (open neighbourhoods),
   (a) What is the meaning of "addition" and "scalar multiplication" are jointly continuous?
   (b) Prove that $B(a, r) = \{x \in X : \|x - a\| < r\}$ is an open set of X, for some $a \in X$ and $r > 0$.
   (c) Prove that $B[a, r] = \{x \in X : \|x - a\| \leq r\}$ is a closed set of X, for some $a \in X$ and $r > 0$.
   (d) Prove that any singleton set $\{a\}$ is a closed set, for some $a \in X$.
5. Which of the following properties are carried over from a normed space X to itself under the translation map $x \mapsto x + a$, for some fixed $a \in X$.
   (a) open sets
   (b) closed sets
   (c) bounded sets
   (d) compact sets
   (e) convergence with limits
   (f) Cauchy sequences
6. In $\mathbb{R}^2$, $f(x, y) = (|x|^{1/2} + |y|^{1/2})^2$. Sketch the closed unit ball $B = \{(x, y) \in \mathbb{R}^2 : f(x, y) \leq 1\}$. Is the closed unit ball convex?
7. Prove that if $x_n \to 0$, then $\|x_n\| \to 0$. Is the converse of the above statement true?
8. Prove that the sum of a norm and a seminorm is a norm.
9. Prove that for each additively invariant and absolutely homogeneous metric on X, there exists a unique norm on X generates it.
10. Prove that a Banach space is finite dimensional iff every subspace of it is closed.