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MA939 Functional Analysis
Problem Sheet - 5

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Answer **ALL** questions.

1. Say true or false : Every nowhere dense subset of a metric space is of the first category. Is the converse of the above statement true?
2. Let A be a subset of a metric space X. Prove that A is a nowhere dense subset of X iff for any open ball $B \subseteq X$ there exists an open ball $B_1 \subseteq B$ such that $B_1 \cap A = \phi$.
3. Prove that every complete metric space X is of the second Baire category. The converse is not true in general. Give an example.
4. Let X be a normed space. Prove that X is Banach iff $\sum y_n$ converges whenever $\|y_n\| \leq \frac{1}{2^n}$ for all n.
5. Prove that a normed space X is strictly convex iff there is some t with $0 < t < 1$ such that $\|tx + (1-t)y\| < 1$ holds whenever $x, y \in X$ with $\|x\| = \|y = \|1$ and $x \neq y$.
6. Prove that a normed space X is strictly convex iff it has the following property : If x and y are nonzero elements in X such that $\|x + y\| = \|x\| + \|y\|$, then $x = cy$ for some real $c > 0$.
7. Consider the linear space $c^{(3)}$ of all sequences $x = (x_n)_{n=1}^{\infty}$ such that $(x_{3k+q})_{k=0}^{\infty}$ converges for $q = 0, 1, 2$. Find the dimension of $c^{(3)}/c_0$.
8. We know that c_0 is a closed subspace of ℓ_∞. Define the quotient norm on the quotient space ℓ_∞/c_0.
9. Let $E_1, E_2, \ldots, E_n$ be subspaces of a vector space X. If codim $E_i = 1$, $i = 1, 2, \ldots, n$, then prove that codim $\cap_i E_i \leq n$.
10. Let M be a finite dimensional subspace of a normed space X. If $x + M \in X/M$, then prove that there exists an element $y \in x + M$ such that $\|y\| = \|x + M\|$.