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National Institute of Technology Karnataka, Surathkal
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MA939 Functional Analysis
Problem Sheet - 8

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Answer **ALL** questions.

1. If A is a separable subset of a normed space X, then prove that $\overline{A}$, $\text{co}(A)$, $\overline{co}(A)$, $\langle A\rangle$ and $[A]$ are all separable.
2. Let X be a normed space. Prove the following:
   (a) Every finite dimensional normed space is separable.
   (b) Prove that X is separable iff its unit sphere S_X is separable.
3. If M is a closed subspace of a separable normed space X, then prove that X/M is separable. Is separability a three space property?
4. Prove that the spaces ℓ_∞ and $L_\infty[0,1]$ are not separable.
5. A normed space is the closed linear hull of one of its compact subsets is called *compactly generated*. Prove the following :
   (a) All finite dimensional normed spaces are compactly generated.
   (b) A normed space is compactly generated iff it is separable.
6. Let $Y = \{(x_n) \in \ell_1 : x_1 = x_3 = x_5 = \cdots = 0\}$ be a subspace of ℓ_1. Then any nonzero continuous linear functional on Y has an infinity of Hahn-Banach extensions.
7. A normed space X is strictly convex iff there is some t with $0 < t < 1$ such that $\|tx + (1-t)y\| < 1$ holds whenever $x, y \in X$ with $\|x\| = \|y\| = 1$ and $x \neq y$.
8. A normed space X is strictly convex iff it has the following property : If x and y are nonzero elements in X such that $\|x + y\| = \|x\| + \|y\|$, then $x = cy$ for some real $c > 0$.
9. State and prove Taylor-Foguel theorem for unique Hahn-Banach extension.
10. Let Y be a subspace of a normed space X and $g : Y \to \mathbb{K}$ is a continuous linear functional for which there exist $f_1, f_2 : X \to \mathbb{K}$, two distinct Hahn-Banach extensions. Prove that the set of all the Hahn-Banach extensions is convex in X^*.