Department of Mathematical and Computational Sciences, NITK, Surathkal
Linear Algebra and Matrices (MA 904)
https://sam.nitk.ac.in/sites/default/MA904-ps-3.pdf

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Problem Sheet 3

1. Let A, B and C be matrices with suitable sizes. Prove that $(AB)C = A(BC)$, the matrix multiplication is associative.

2. Choose the only B (3 by 3 matrix) so that for every matrix A,

   (a) $BA = 4A$.

   (b) $BA = 4B$.

   (c) BA has rows 1 and 3 of A reversed and row 2 unchanged.

   (d) All rows of BA are the same as row 1 of A.

3. What rows or columns or matrices do you multiply to find

   (a) the third column of AB?

   (b) the first row of AB?

   (c) the entry in row 3, column 4 of AB?

   (d) the entry in row 1, column 1 of CDE?

4. If you multiply a *northwest matrix* A and a *southeast matrix* B, what type of matrices are AB and BA? "Northwest" and "southeast" mean zeros below and above the antidiagonal going from $(1, n)$ to $(n, 1)$.

5. *Elimination for a* 2 *by* 2 *block matrix:* When $A^{-1}A = I$, multiply the first block row by CA^{-1} and substract from the second row, to find the "*Schur complement*" S:

$$\begin{pmatrix} I & 0 \\ -CA^{-1} & I \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & B \\ 0 & S \end{pmatrix}.$$

6. Invent a 3 by 3 **magic matrix** M with entries $1, 2, \ldots, 9$. All rows and columns and diagonals add to 15. The first row could be $8, 3, 4$. What is M times $(1, 1, 1)$? What is the row vector [1 1 1] times M?

7. Find all matrices $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ that satisfy $A \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} A$.