Department of Mathematical and Computational Sciences, NITK, Surathkal
Linear Algebra and Matrices (MA 904)
https://sam.nitk.ac.in/sites/default/MA904-ps-2.pdf

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Problem Sheet 2

1. Find E^2, E^8 and E^{-1} if $E = \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$.

2. Let $P_{n\times n}$ be any permutation matrix. Prove that $P^m_{n\times n} = I_{n\times n}$ for some m.

3. Find LU factorization of the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{pmatrix}$.

4. Find LDU factorization of the matrix $\begin{pmatrix} 3 & -1 & 2 \\ -3 & -2 & 10 \\ 9 & -5 & 6 \end{pmatrix}$.

5. Given A, find the LU factorization of A^TA and AA^T and compare the factorizations.

Inverse

6. Let $x = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$ and $A = \begin{pmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{pmatrix}$. Is x in the column space of A? why or why not?

7. Invert the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ by the Gauss-Jordan method.

8. Let $A = \begin{pmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{pmatrix}$. Find the third column of A^{-1} with out computing the other columns.

9. Let A be an invertible $n \times n$ matrix, and let B be an $n \times n$ matrix. If $[A \quad B]$ is row equivalent of $[I \quad X]$, what is the relation between X, A and B?

10. Let $A = \begin{pmatrix} 2 & 5 \\ -3 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -5 \\ 3 & k \end{pmatrix}$. What value(s) of k, if any, will make $AB = BA$?

11. Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? What about n vectors in $\mathbb{R}^m$ when in n is less than m?

12. Let A be a 3×4 matrix, let y_1 and y_2 be vecotrs in $\mathbb{R}^3$, and let $w = y_1 + y_2$. Suppose $y_1 = Ax_1$ and $y_2 = Ax_2$ for some vectors x_1 and x_2 in $\mathbb{R}^4$. What fact allows you to conclude that the system $Ax = w$ is consistent?

13. Give all the 2×2 matrices A such that $A^2 = I$.

14. Let $S = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. Compute the least positive integer k such that S^k is the zero matrix.