Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Numerical Analysis - MA 704
Problem Sheet 8

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1. Using the Richardson's extrapolation limit, find $y'(0.05)$ to the function

$$y = -\frac{1}{x}$$

with $h = 0.0128, 0.0064, 0.0032$. Use the formula $y'(x) \approx \frac{y(x+h)-y(x-h)}{2h}$.

2. From the following data, estimate the value of $\int_1^5 \log x dx$, using Simpson's 1/3 rule. Also obtain the value of h so that the value of the integral will be accurate upto five decimal places.

x	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
$\log x$	0.0000	0.4055	0.6931	0.9163	1.0986	1.2528	1.3863	1.5041	1.6094

3. Using Adams-Moulton predictor-corrector method find the solution of the initial value problem at $x = 0.8$ given $y' = y - x^2, y(0) = 1.0000 y(0.2) = 1.21859, y(0.4) = 1.46813$. Use Runge-Kutta method of order 4 to compute $y(0.6)$.

4. Solve $y'' + xy = 0, y(0) = 1, y'(0) = 0$, using Numerov method Use Taylor series method of order 4 to compute the missing values.

5. Given $u_{xx} + u_{yy} = x^2 - 1, |x| \leq 1, |y| \leq 1$ with $u = 0$ on the boundary of the square. Formulate the nine point difference scheme with mesh size $h = 0.5$ in both directs.

6. Derive the Crank Nicolson formula to solve $u_t = u_{xx}$. Use the method to find the numerical solution of the equation after one time step subject to the initial condition $u(x, 0) = 1, 0 < x < 1$ and the boundary conditions $u(0, t) = u(1, t) = 0, t \geq 0$. Write the difference scheme to compute the solution by taking $h = 1/4$ and $k = 1/32$.

7. Solve the wave equation $u_{tt} = u_{xx}, 0 \leq x \leq 1$, subject to the initial conditions $u(x, 0) = x^2, u_t(x, 0) = 1$ and the boundary conditions $u(0, t) = 0, u(1, t) = 1 + t, t > 0$ by taking $h = k = 0.2$. Compute the solution for the first time step.

8. Obtain the cubic spline approximation in the interval (2,3) for the function given below.

x	1	2	3	4
y	1	5	11	8

9. (a) Suppose that the function $f(x)$ is defined for the $(n + 1)$ distinct points $x_0, x_1, \ldots, x_n$ which are not necessarily equidistant. Suppose that $f(x)$ is represented by an nth degree polynomial. Obtain an expression for the derivative of $f(x)$ at any tabulated point. Also obtain the expression for the error of approximation of the derivative at the tabulated point.

   (b) The values of $f(x) = \sqrt{x}$ are given in the following table. Use the data to estimate $f'(0.6)$

x	0.5	0.6	1.0
$f(x)$	0.70711	0.77460	1.00000

10. Given $y' = -5y + 3z, y(0) = 2$, $z' = -3y - 5z, z(0) = 2$. Apply Runge-Kutta method of order 4, to compute the solution of the system at $x = 0.1$ with step length $h = 0.1$.

11. Given $y'' - y = x, y(0), y'(0) = -2$. Apply Numerov method to compute the value of y at $x = 0.2$ with step length $h = 0.2$.

12. Write down the finite difference equations to solve the mixed boundary value problem for the Poisson equation $\nabla^2 u = 2(x^2 + y^2)$ in the region bounded by the lines $x = 0, y = 0, x = 3$ and $y = 3$ for the boundary conditions $u = 0$ on $x = 0, y = 0$

$$u_x = 6y^2 \text{ on } x = 3u = 9x^2 \text{ on } y = 3$$

Take $h = k = 1$ and use the five point formula.

13. (a) Derive the Crank-Nicolson formula to solve the heat conduction equation $u_t = u_{xx}$.

    (b) Apply the above formula to the above equation subject to the initial conditions $u(x, 0) = \sin \pi x, 0 \leq x \leq 1$ and the boundary conditions $u(0, t) = u(1, t) = 0$. Choose $h = 1/3$, $k = 1/36$ and compute the solution at time level 1.

14. Find the solution of the vibrating string problem $u_{tt} = u_{xx}$ under the conditions

$$\begin{aligned} u(x,0) &= x^2 \\ u_t(x,0) &= 2x \\ u_x(0,t) &= 2t \\ u(1,t) &= (1+t)^2 \end{aligned}$$

with $h = 1/3, k = 1/6$ for two time levels.