

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Numerical Analysis - MA 704
Problem Sheet 5

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1. The function $y = \sin x$ is tabulated in the scheme below. Find the derivative at the point $x = 1$. Also find the error.

x	0.7	0.8	0.9	1.0	1.1	1.2	1.3
y	0.644218	0.717356	0.783327	0.841471	0.891207	0.932039	0.963558

2. The values of $f(x) = \sqrt{x}$ are given in the following table. Use the data to estimate $f'(0.6)$.

x_i	0.5	0.6	1.0
$f(x_i)$	0.70711	0.77460	1.00000

Obtain a theoretical bound on the error in this estimate and compare this with the actual value.

3. Consider the table of data for $(f(x) = \frac{1}{x})$.

x	1.0	1.5	2.0	2.5	3.0
$f(x)$	1.000	0.667	0.500	0.400	0.333

Find values for $f'(x)$ and $f''(x)$ at $x = 1.5, 2.0$ & 2.5 from the cubic spline functions that approximate $f(x)$.

4. The following table is for $f(x) = 1 + \log x$. Determine $f'(x)$ at $x = 0.15, 0.19$ and 0.23 using
- (a) one term (b) two terms (c) three terms.

x	0.15	0.17	0.19	0.21	0.23	0.25	0.27	0.29	0.31
y	0.1761	0.2304	0.2788	0.3222	0.3617	0.3979	0.4314	0.4624	0.4914

Write expressions for the errors in each computation. From these expressions find upper and lower bounds for each of these computations.

5. Estimate $f''(1)$ for the function $f(x) = \sin 4x$ using

$$f_1'' \approx \frac{1}{h^2}(f_2 - 2f_1 + f_0),$$

$h = 0.05, 0.04, 0.03, 0.02$ and 0.01 work to 5 decimals.