Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal
Numerical Analysis - MA 704
Problem Sheet 2

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1. Use Newton-Raphson method to obtain a root, to 3 decimal places, of the following equations.
   (a) $x^3 - 2x - 5 = 0$
   (b) $x^3 + 5x + 1 = 0$
   (c) $x^3 - 5x + 3 = 0$
   (d) $\sin x = 1 - x$
   (e) $\tan x = 4x$
   (f) $x^4 + x^2 - 80 = 0$
   (g) Determine the multiplicity of the root $r = 1$ of the polynomial equation $P(x) = x^5 - 2x^4 + 4x^3 - x^2 - 7x + 5 = 0$.

2. Solve the following systems of nonlinear equations by Newton-Raphson Method.
   (a) $x^2 + y = 11; y^2 + x = 7$
   (b) $x^3 = y + 100; y^3 = x + 150$
   (c) $x^2 = 3xy - 7; y = 2(x + 1)$
   (d) The system of equations
   $$y\cos(xy) + 1 = 0 \qquad \text{and} \qquad \sin(xy) + x - y = 0$$
   has one solution close to $x = 1, y = 2$. Calculate this solution correct to 2 decimal places.
   (e) Find the intersection between the curves $y = e^x - 2$; $y = \log(x + 2)$ to four decimals $(x > 0)$.
   (f) The curves $3x^2 - 2xy + 5y^2 - 7x + 6y - 10 = 0$ and $2x^2 + 3xy - y^2 - 4 = 0$ have one intersection point in the first quadrant. Find its coordinates to four places.

3. Use Mullers method to find a root of the following equations.
   (a) $x^3 - x - 1 = 0$
   (b) $x^3 - x^2 - x = 0$.

4. Perform 2 iterations with the Muller method for the following equations
   (a) $x^3 - \frac{1}{2} = 0, x_0 = 0, x_1 = 1, x_2 = \frac{1}{2}$
   (b) $\log x - x + 3 = 0, x_0 = \frac{1}{4}, x_1 = \frac{1}{2}, x_2 = 1$.

5. Apply the Newton-Raphson method with $x_0 = 0.8$ and the Muller method with $x_0 = 0.6, x_1 = 0.8, x_2 = 1.2$ to the equation
   $$f(x) = x^3 - x^2 - x + 1 = 0$$
   and verify that the convergence is only of first order in each case. Then apply the Newton-Raphson method with $p = 2$ and verify that the convergence of second order.