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Linear Algebra and Matrices (MA 204)

https://sam.nitk.ac.in/sites/default/MA204-ps-5.pdf

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Problem Sheet 5

1. For what values of a, b, c the following system will have no solution, one solution and infinite solutions

$$\begin{aligned} 4x + y + az &= 7 \\ -x + by + z &= 11 \\ -2x + y - 3z &= c. \end{aligned}$$

2. Give a basis for each of the four fundamental subspaces associated to the following matrix

$$A = \begin{pmatrix} 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}.$$

3. For any values of a, b and c, find the determinant of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{pmatrix}$.

4. Find the values of a for which the planes $ax - y + z = 1$ and $3ax + ay - 2z = 5$ are perpendicular.

5. Suppose $A = \begin{pmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \\ (row & 3 & of & A) \end{pmatrix}$ has reduced echelon form $R = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

   (a) What can you say about row 3 of A?

   (b) What are the numbers a and b?

   (c) Describe the nullspace of A.

6. State whether the following are true or false and give a reason or a 2×2 or 3×3 counter example. Here $|A|$ denotes the determinant of matrix A.

   (a) The determinant of $A + I = (\texttt{det}A) + 1$.

   (b) The determinant of $A^4 = |A|^4$.

   (c) The determinant of $4A = 4|A|$.

   (d) The determinant of $A = |A|$.

   (e) If A is not invertible, then AB is not invertible.

   (f) The determinant of $AB = |A||B|$.

   (g) AB and BA have the same determinant.